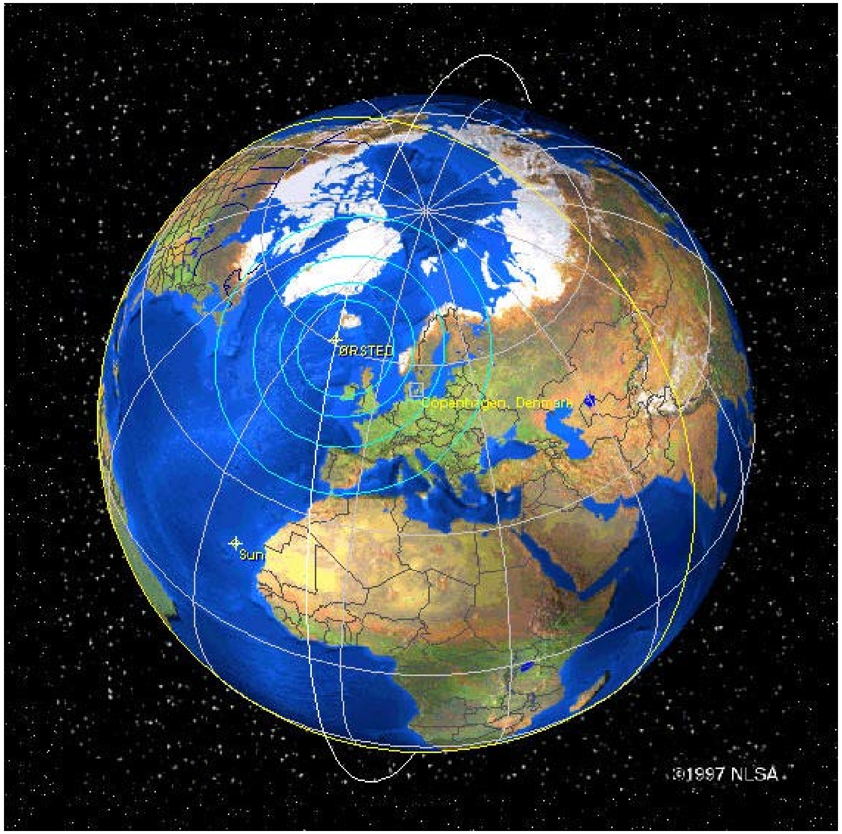
Danish Space Research Institute

Danish Small Satellite Programme

DTU Satellite Systems and Design Course
Orbital Mechanics



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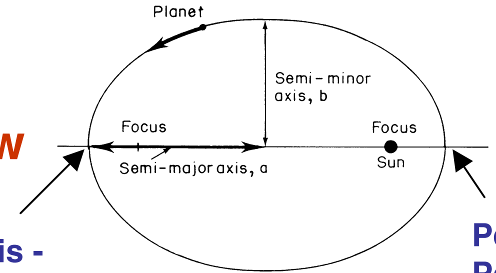
Planetary and Satellite Orbits

♦ Johannes Kepler (1571 - 1630)

- **Discovered by the precision mesurements of Tycho Brahe that the Moon and the planets moves around in elliptical orbits**
- **Harmonia Mundi 1609, Kepler's 1st og 2nd law of planetary motion:**
- **1st Law: The orbit of a planet ia an ellipse with the sun in one focal point.**
- **2nd Law: A line connecting the sun and a planet sweeps equal areas in equal time intervals.**
- **1619 came Keplers 3rd law:**
- **3rd Law: The square of the planet's orbit period is proportional to the mean distance to the sun to the third power.**

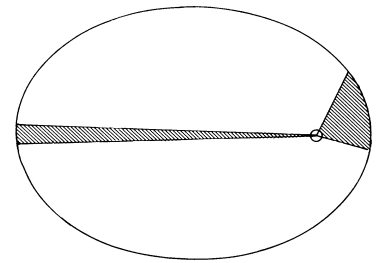
1st Law



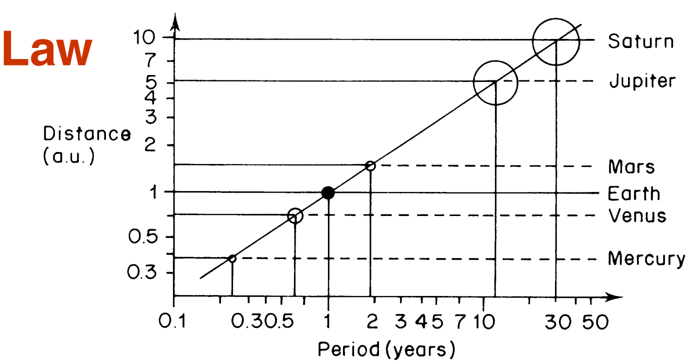
Apoapsis - Aphelion

Periapsis - Perihelion

2nd Law



3rd Law



Newton's Laws

♦ Isaac Newton (1642 - 1727)

- Philosophiae Naturalis Principia Mathematica 1687
- 1st Law: The law of inertia
- 2nd Law: Force = mass x acceleration
- 3rd Law: Action og reaction
- The law of gravity:

$$F = \frac{GMm}{r^2}$$

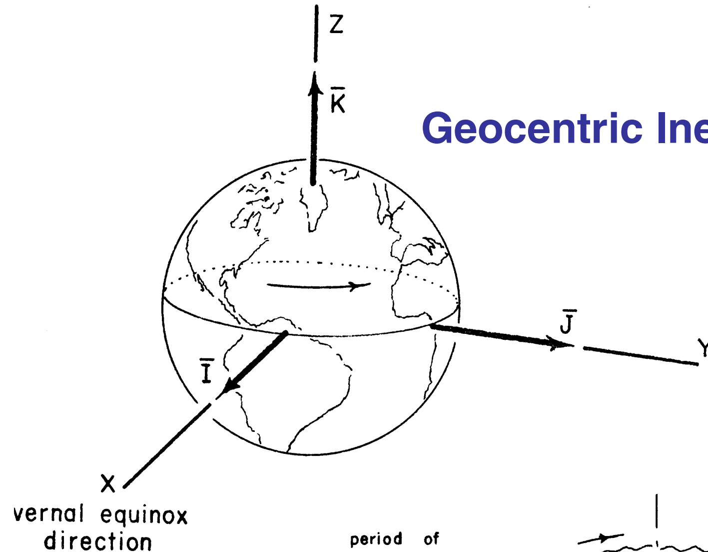
F	Gravitational force between two bodies
G	The universal gravitational constant: $G = 6.670 \bullet 10^{-11}\ Nm^2kg^{-2}$
M	Mass af one body, e.g. the Earth or the Sun
m	Mass af the other body, e.g. the satellite
r	Separation between the bodies

G is difficult to determine precisely enough for precision orbit calculations.

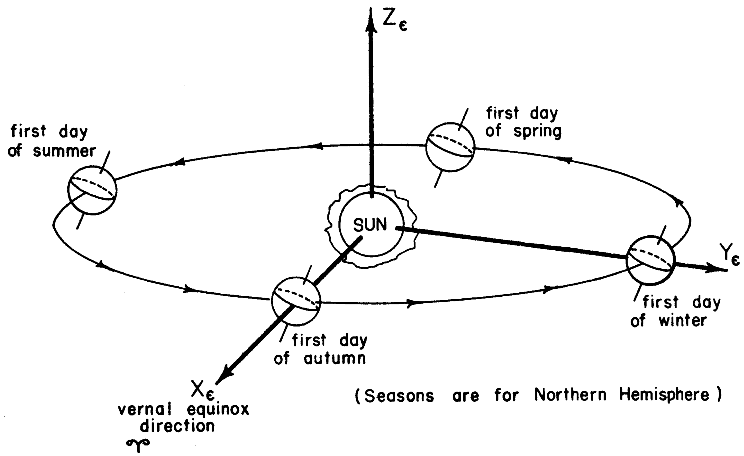
GM_{Earth} can be determined to great precision: $GM_{Earth} = \mu = 3.986004418 \cdot 10^{14}\ m^3/s^2$

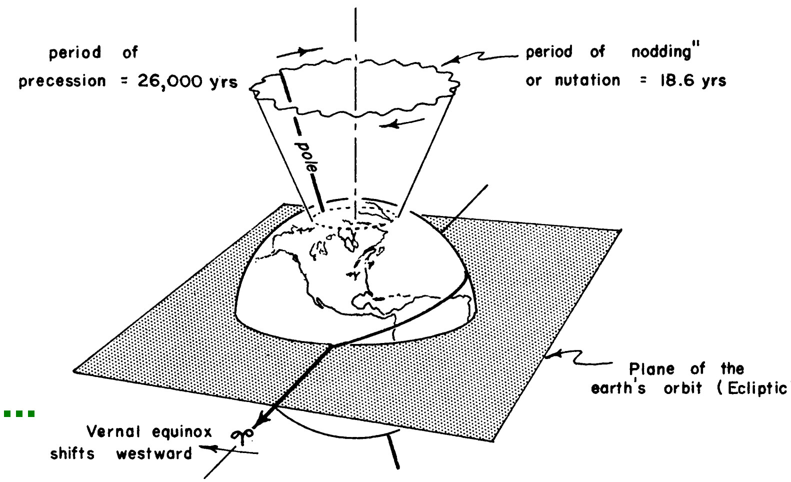
Coordinate Systems

Geocentric Inertial System



Heliocentric Inertial System

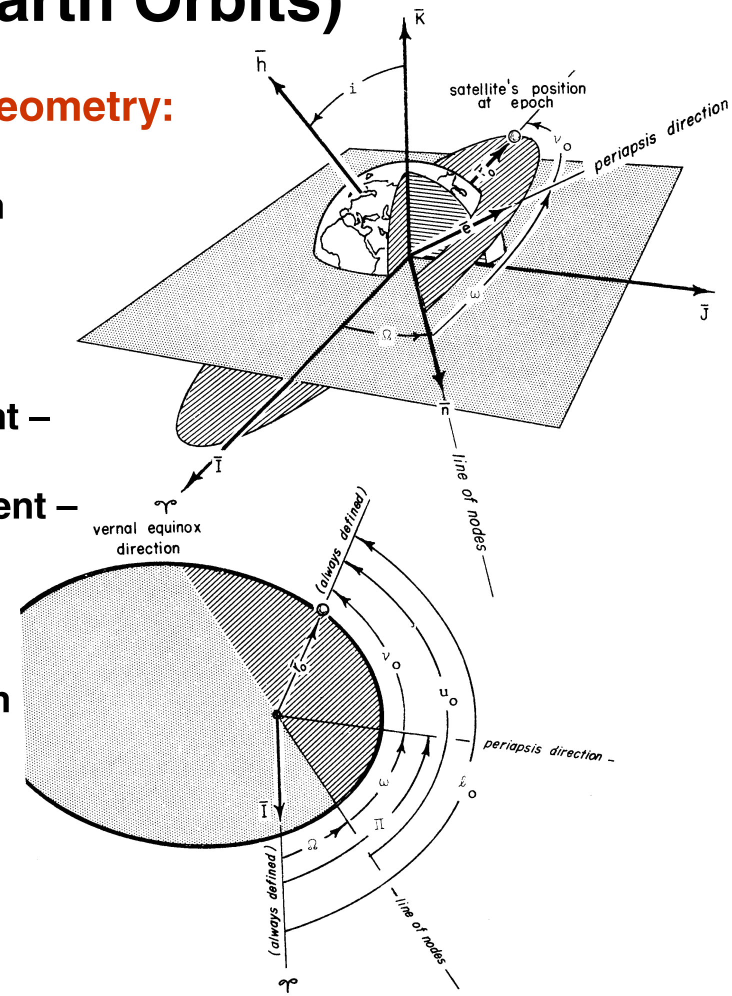




The reference changes with time .....

50.2786" westerly drift of the Vernal Equinox per year

Kepler Elements, Orbital Elements (Earth Orbits)



Five orbital elements are neded to determine the orbit geometry:

a **Semi-Major Axis**
Determines the size of the orbit and the period of revolution

e **Eccentricity** – Determines how elongated the ellipse is

i **Inclination**
Angle between the equator and orbital planes
$0° \leq i \leq 90°$: The satellite has an easterly velocity component – prograde orbit
$90° \leq i \leq 180°$: The satellite has a westerly velocity component – retrograde orbit

Ω **Right Ascension of Ascending Node - RAAN**
Angle from Vernal Equinox to the point where the orbit intersects the equatorial plane passing from South to North (the Ascending Node).

ω **Argument of Periapsis / Perigee**
Angle Between the line of nodes (the line between the ascending and descending nodes) and periapsis / perigee

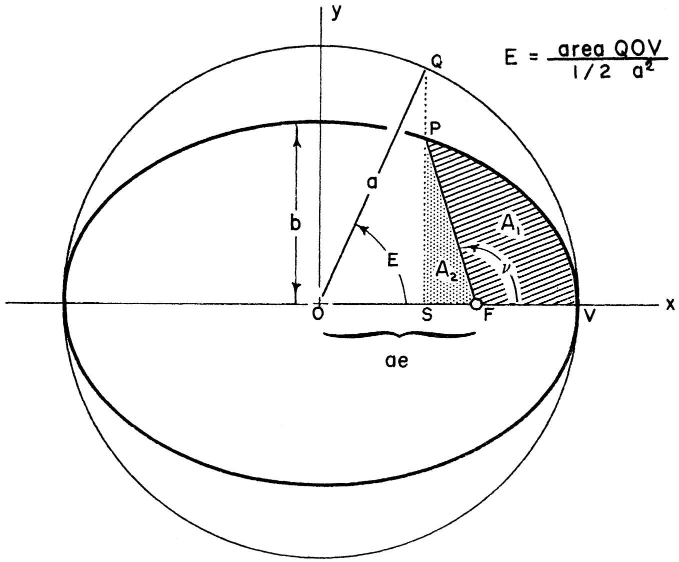
Two more elements are needed to calculate the position of the satellite at any time:

T_0 **Epoch Time** – Reference time for orbital elements and start of orbit propagation

ν_0 **True anomaly at Epoch** – Angle from periapsis / perigee to the satellite position at T_0

Normally the parameter "Mean Motion" is used instead of the semi-major axis in orbital calculations:

Mean Motion: $n := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{\mu}{a^3}}$ **Orbit period:** $Tp := 2\cdot\pi\cdot\sqrt{\left(\frac{a^3}{\mu}\right)}$



Normally the parameter Mean Anomaly instead true anomaly in an orbital parameter set:

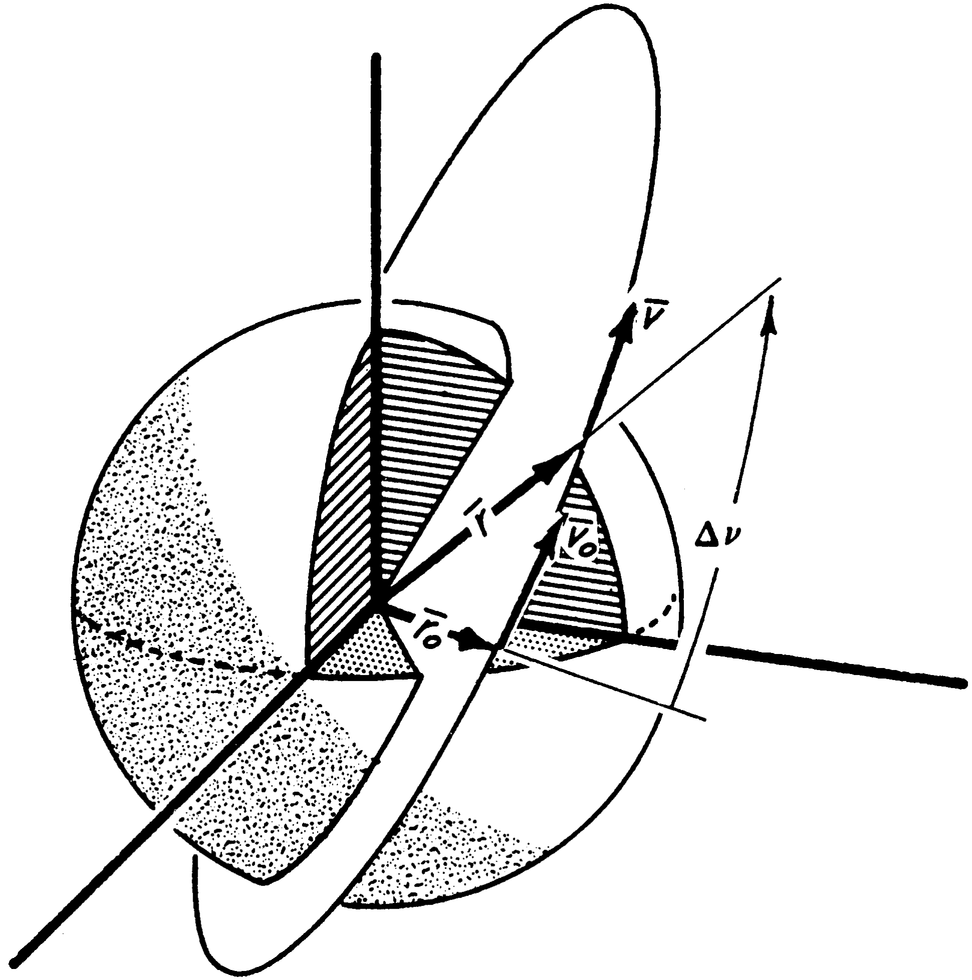
$$M - M_0 = n\,(t - T_0) = E - e \sin E$$ **(Kepler's equation)**

where E is the Eccentric Anomaly

Kepler’s problem

Given: $\underline{\mathbf{r}}_0$, $\underline{\mathbf{v}}_0$ at time T_0

Find: $\underline{\mathbf{r}}$, $\underline{\mathbf{v}}$ at time t



Perturbations

The fact that the Earth is not a sphere but an ellipsoid causes the orbit of a satellite to be perturbed

Geopotential function:

$$\Phi(r,L) = (\mu/r) \bullet [1 - \sum J_n(R_E/r)^n P_n(\sin L)]$$

where:

L is latitude

R_E is the Earth radius at Equator

P_n is the Legendre polynomials

J_n are dimensionless coefficients:

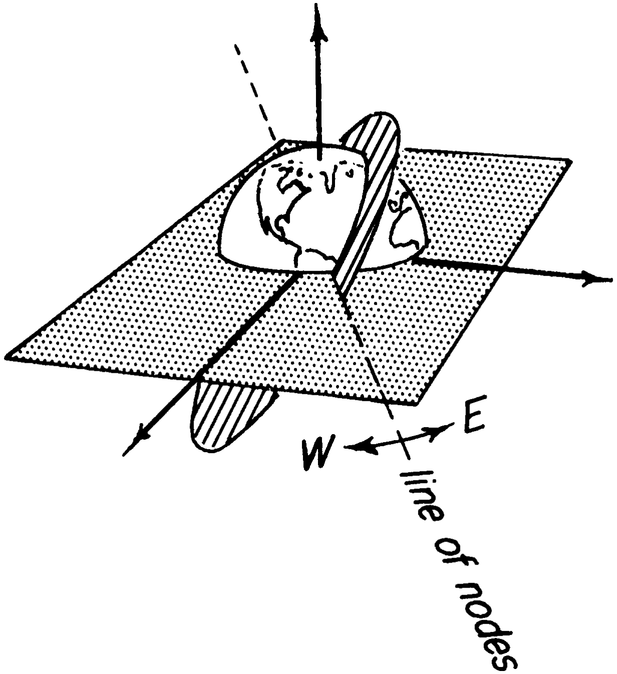
- $J_2 = 0.00108263$
- $J_3 = -0.00000254$
- $J_4 = -0.00000161$
- ...

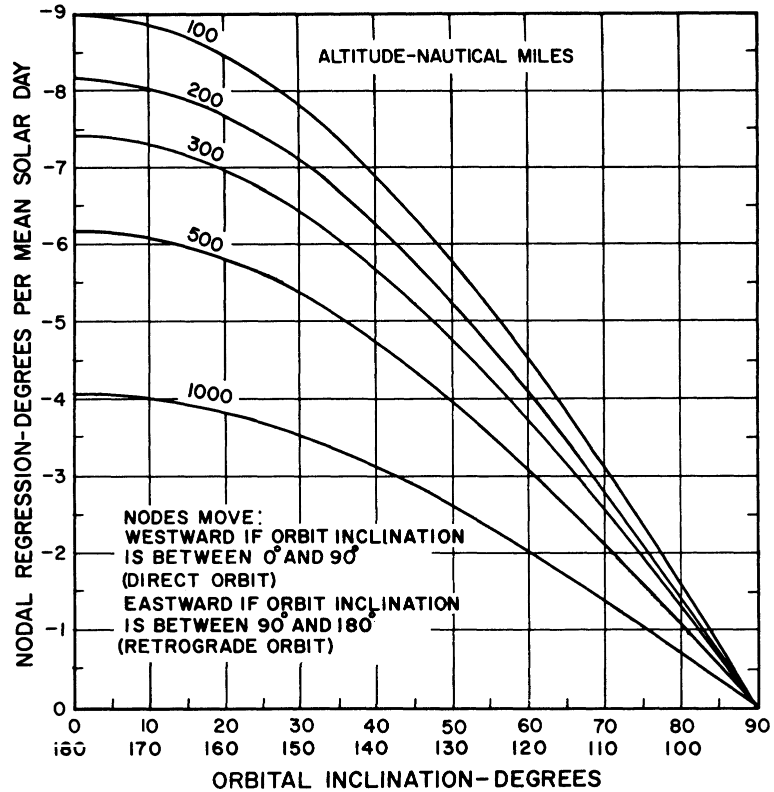
Orbit	Effect of J_2 (Eqs. 6-19, 6-20) (deg/day)	Effect of Moon (Eqs. 6-14, 6-16) (deg/day)	Effect of Sun (Eqs. 6-15, 6-17) (deg/day)
Shuttle	a = 6700 km, e = 0.0, i = 28 deg		
$\Delta\Omega$	–7.35	–0.000 19	–0.000 08
$\Delta\omega$	12.05	0.002 42	0.001 10
GPS	a = 26,600 km, e = 0.0, i = 60.0 deg		
$\Delta\Omega$	–0.033	–0.000 85	–0.000 38
$\Delta\omega$	0.008	0.000 21	0.000 10
Molniya	a = 26,600 km, e = 0.75, i = 63.4 deg		
$\Delta\Omega$	–0.30	–0.000 76	–0.000 34
$\Delta\omega$	0.00	0.000 00	0.000 00
Geosynchronous	a = 42,160 km, e = 0, i = 0 deg		
$\Delta\Omega$	–0.013	–0.003 38	–0.001 54
$\Delta\omega$	0.025	0.006 76	0.003 07

$\Delta\Omega$ is the drift of the ascending node

$\Delta\omega$ is the drift of periapsis / perigee

Drift of Ascending Node

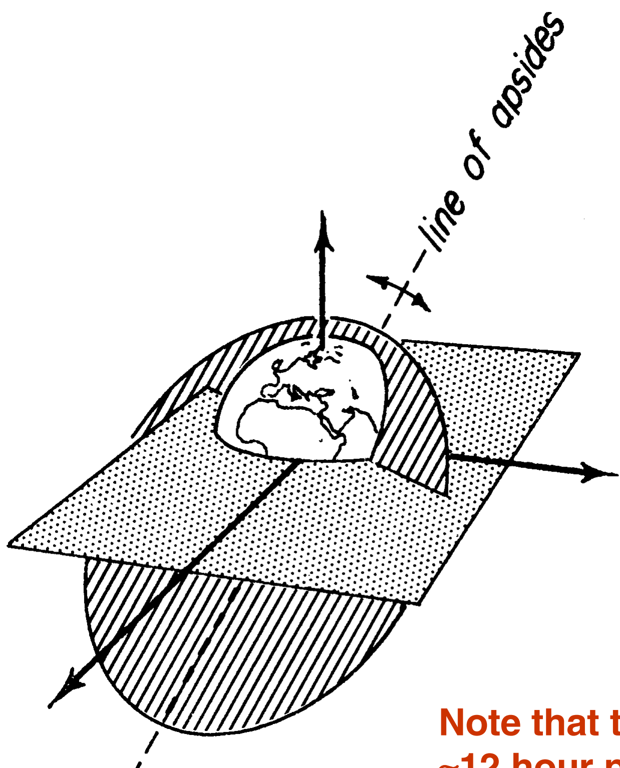


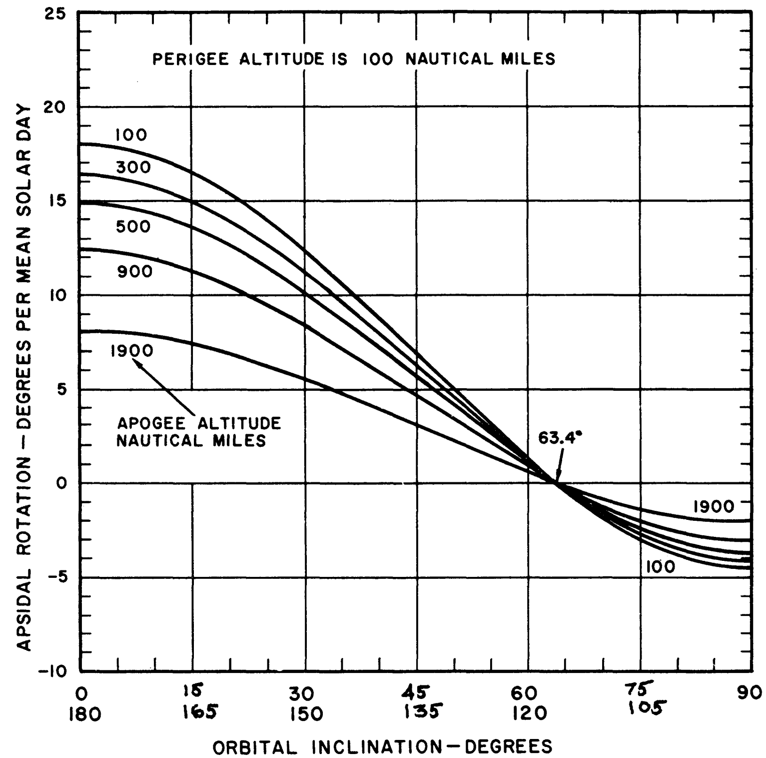


$$\mathbf{NodalDriftRate}(a, i, e) := -2.06474 \cdot 10^{14} \cdot \left(\frac{a}{1 \cdot km}\right)^{\frac{-7}{2}} \cdot \cos(i) \cdot \left(1 - e^2\right)^{-2} \cdot \frac{deg}{day}$$

The Sun moves easterly by 0.9856° per day. An orbit exhibiting the same drift of the ascending node is said to be sun-synchronous (Helio-synchronous). This requires i > 90° i.e. a retrograde orbit. The satellite crosses the Equator at the same local solar time every orbit.

Drift of Periapsis / Perigee

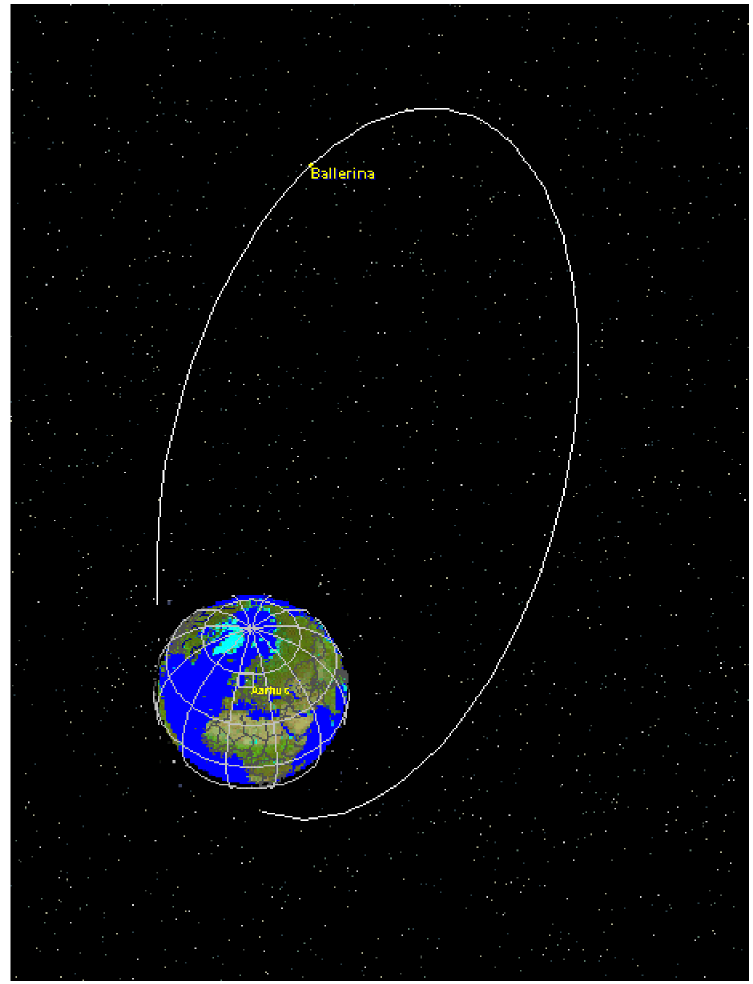




Note that the drift is zero at i = 63.435°. This type of orbit with e ≈ 0.75, ≈12 hour period and apogee on the Northern hemisphese is denoted Molniya orbit and is used by Russian communication satellites

$$\mathrm{ArgPerigeeDriftRate}(a, i, e) := 1.03237 \cdot 10^{14} \cdot \left(\frac{a}{1 \cdot \mathrm{km}}\right)^{\frac{-7}{2}} \cdot \left(4 - 5 \cdot \sin(i)^2\right) \cdot \left(1 - e^2\right)^{-2} \cdot \frac{\mathrm{deg}}{\mathrm{day}}$$

Molniya Orbit



Drift of periapsis / perigee is zero at i = 63.435°.

This type of orbit with e ≈ 0.75, ≈12 hour period and apogee on the Northern hemisphese is denoted Molniya orbit and is used by Russian communi-cation satellites

The advantage is that the apogee stays at the Northern hemisphere and is high in the sky for around 8 hours

A geostationary satellite far North (or South) is near the horizon and is easily blocked by mountains or buildings.

Other Orbits

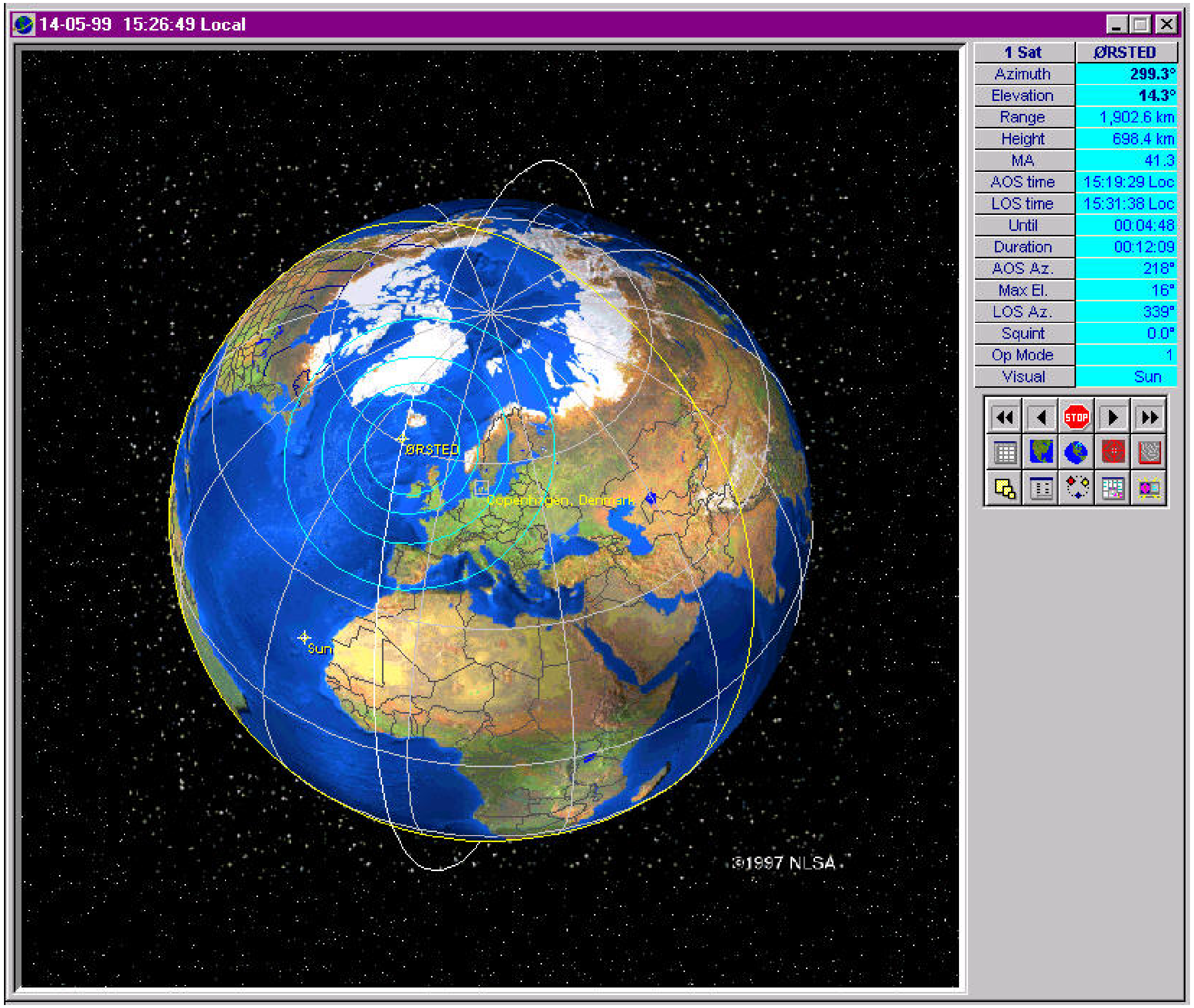
Geostationary Orbit (GEO or GSO).
Circular orbit with inclination = 0º, semi-major axis = 42164.172 km, h = 35768 km
Period = 23 h 56 m 4.0954 s

Geostationary Transfer Orbit (GTO)
Elliptical Orbit with inclination ≈ 7º (from Kourou), ≈ 28.5º (from Cape Canaveral)
Perigee ≈ 500 – 600 km, Apogee = 35768 km)

Low Earth Orbit (LEO)
Orbits in the altitude range 250 – 1500 km

Medium Earth Orbit (MEO)
Orbits in the altitude range 10000 – 25000 km

Software for Orbit Calculations



NOVA for Windows 32

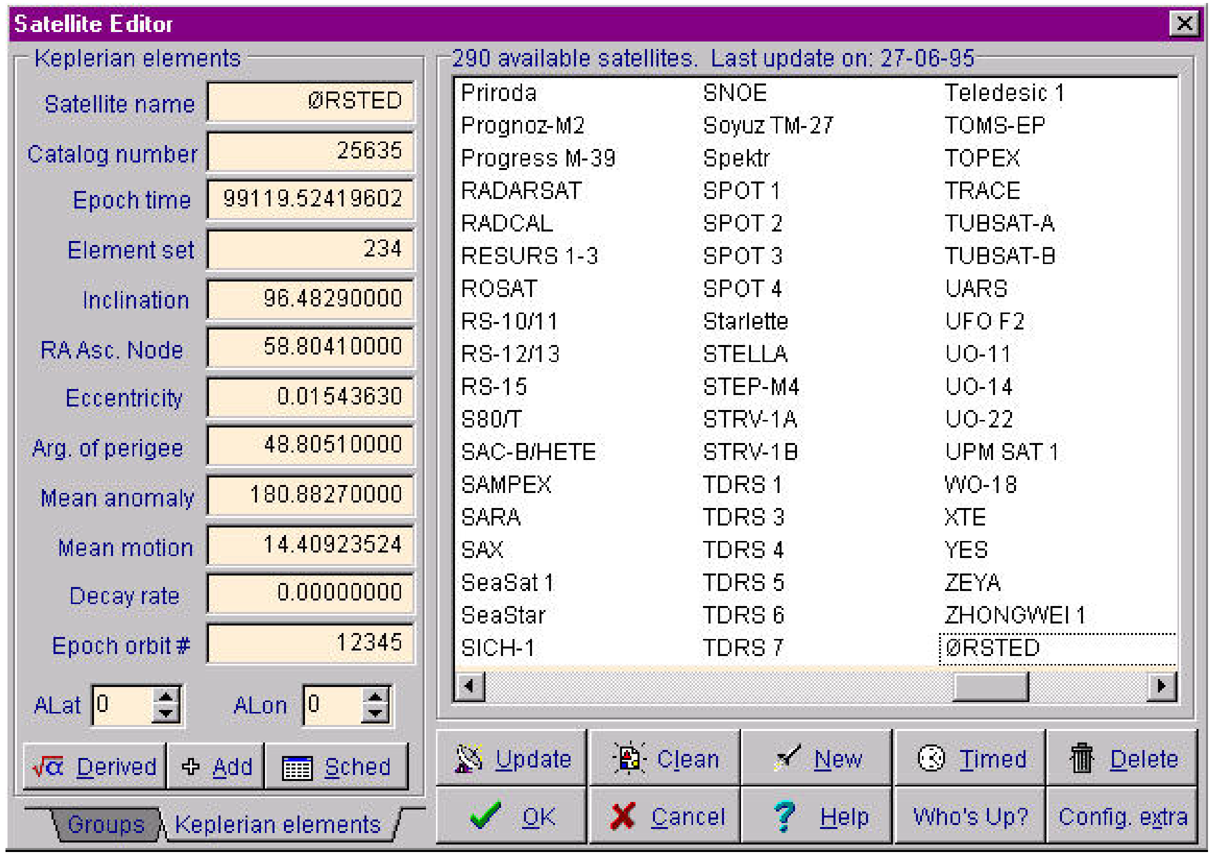
Shareware at USD 59.95

Download from
http://www.nlsa.com

Free demo which may be registered for legal use.

DSRI

NOVA for Windows
Orbital Elements



Orbital Elements in NASA TLE (Two-Line Elements) format

```
        ØRSTED = Name of spacecraft (Max. 11 characters)
Col. No. ....:....1....:....2....:....3....:....4....:....5....:....6....:....7
         1234567890123456789012345678901234567890123456789012345678901234567890
Legend   L AAAAAU YYNNNPPP XXDDD.FFFFFFFF  CCCCCCCC  UUUUUUU  VVVVVVV E SSSSZ
TLE 1    1 25635U 99008B   99132.50239417 +.00000000 +00000+0 +00000+0 1 12344
Col. No. ....:....1....:....2....:....3....:....4....:....5....:....6....:....7
         1234567890123456789012345678901234567890123456789012345678901234567890
Legend   L AAAAA III.IIII RRR.RRRR EEEEEEE PPP.PPPP NNN.NNNN MM.MMMMMMMMKKKKKZ
TLE 2    2 25635  96.4836 68.6614  0153506   8.1032 183.1749 14.40955463 11246
```

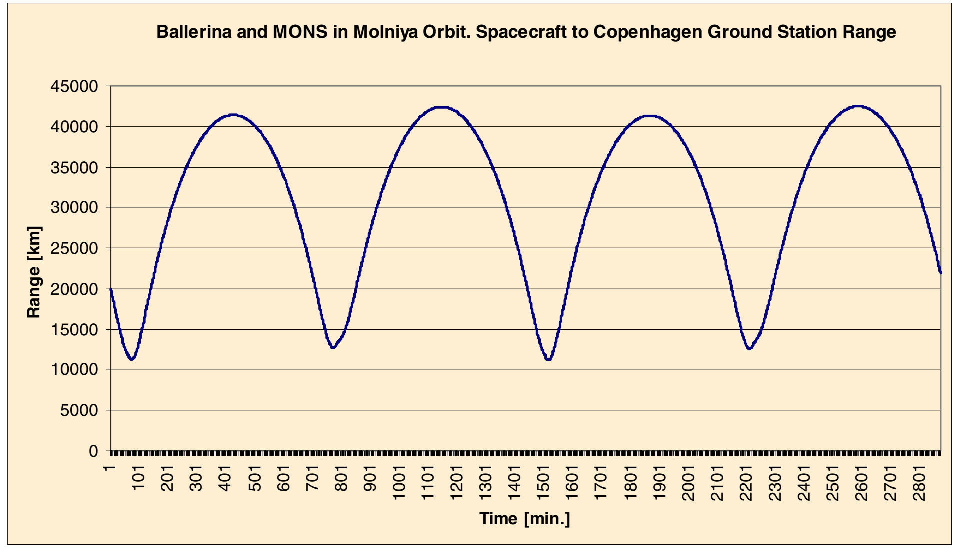
TLE Data Line 1
L: Line Number
A: Catalog Number
U: Unclassified (C: Classified)
Y: Last Two Digits of Daunch Year
N: Launch Number of the Year
P: Piece of Launch
X: Last two digits of Epoch Year
D: Day number in Epoch Year
F: Fraction of day in Epoch Time
C: Decay Rate (First Time Derivative of the Mean Motion divided by 2)
U: Second Time Derivative of Mean Motion divided by 6. (Blank if N/A)
V: BSTAR drag term if GP4 general perturbation theory was used.
Otherwise, radiation pressure coefficient.
E: Ephemeris Type (=1 normally)
S: Element Set Number
Z: Checksum

TLE Data Line 2
L: Line Number
A: Catalog Number
I: Inclination
R: Right Ascension of Ascending Node
E: Eccentricity, assuming implicitly a decimal point at left
P: Argument of Perigee
N: Mean Anomaly
M: Mean Motion
K: Orbit Number
Z: Checksum

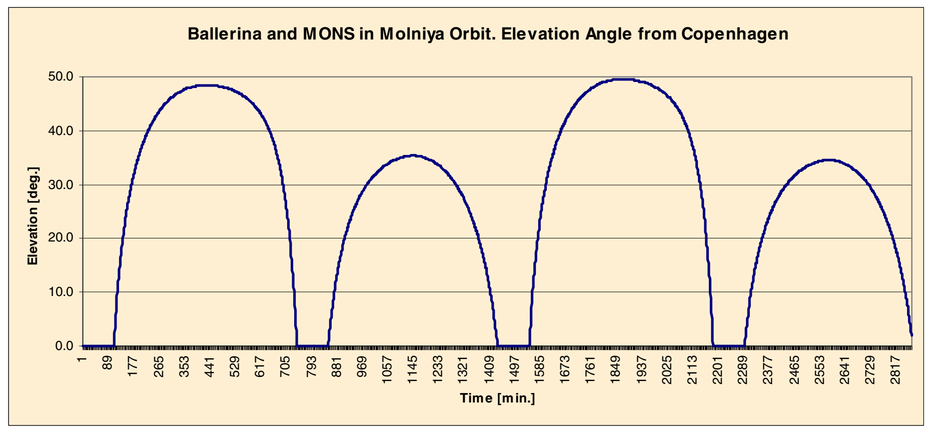
Use of Orbit Calculations for Mission Planning

Eksempel: Rømer mission in Molniya orbit

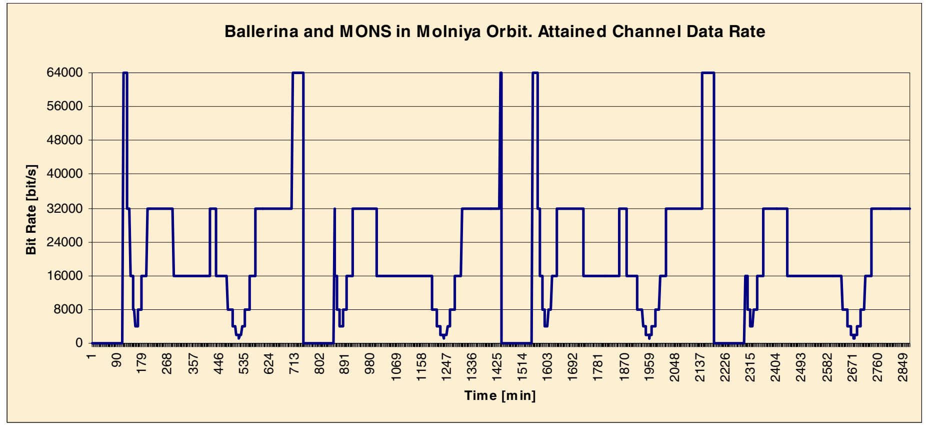
Separation satellite – ground station versus time



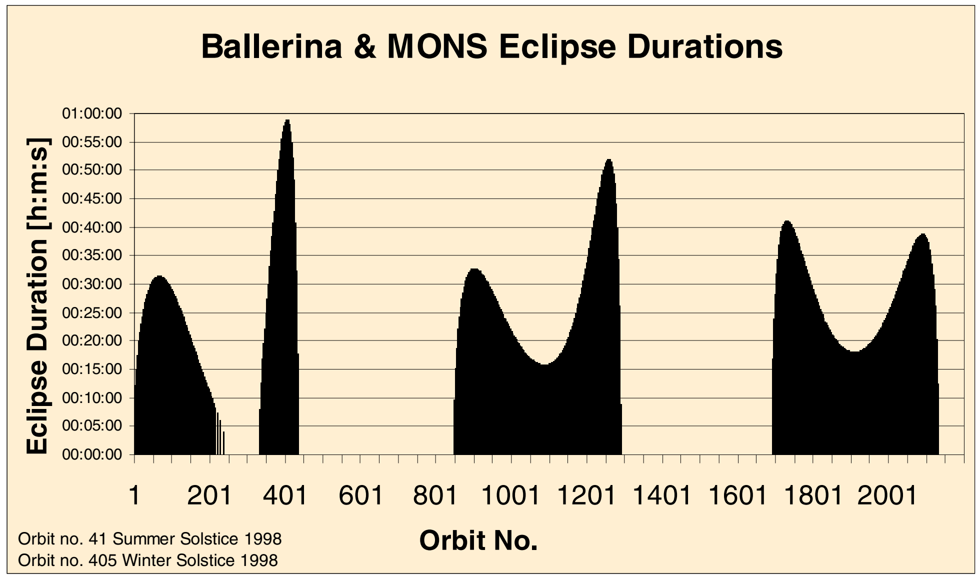
Elevation at ground station versus time



Downlink bit-rate versus time



Satellitten in eclipse (in Earth's shadow).
Max. approx. 1 hour at winter solstice



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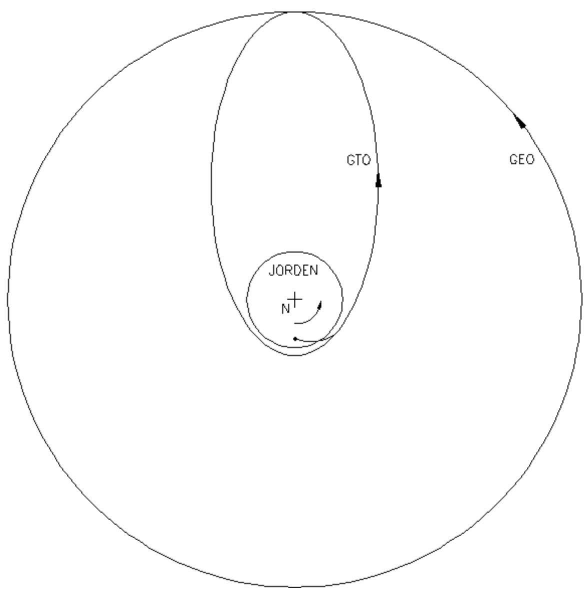
Use of Orbit Calculations for Mission Planning

Calculation of delta-V needed to change apogee for Geostationary Transfer Orbit (GTO) to reach the Moon



Initial Orbit Parameters (GTO)

Apogee Height & Radius	$ha1 := 36786 \cdot km$	$ra1 := ha1 + Rem$
Perigee Height & Radius	$hp1 := 500 \cdot km$	$rp1 := hp1 + Rem$
Inclination	$i1 := 7 \cdot deg$	
Semi-Major Axis	$a1 := \frac{ra1 + rp1}{2}$	$a1 = 25014.001 \circ km$
Eccentricity	$e1 := \frac{ra1}{a1} - 1$	$e1 = 0.725314$
Orbit Period	$Tp1 := 2 \cdot \pi \cdot \sqrt{\frac{a1^3}{\mu}}$	$Tp1 = 10.937 \circ hr$
Mean Motion	$n1 := \frac{2 \cdot \pi}{Tp1}$	$n1 = 2.194461949 \circ \frac{rev}{day}$
Specific orbital energy	$E1 := \frac{-\mu}{2 \cdot a1}$	$E1 = -7.968 \cdot 10^6 \circ \frac{J}{kg}$
Velocity at Apogee	$va1 := Velo(E1, ra1)$	$va1 = 1592.802 m \cdot s^{-1}$
Velocity at Perigee	$vp1 := Velo(E1, rp1)$	$vp1 = 10004.444 m \cdot s^{-1}$

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Orbit Calculations, Continued

Final Orbit Parameters (Raised Apogee)

Apogee Height & Radius	$ha2 := 384400\,km$	$ra2 := ha2 + Rem$
Perigee Height & Radius	$hp2 := hp1$	$rp2 := hp2 + Rem$
Inclination	$i2 := i1$	
Semi-Major Axis	$a2 := \frac{ra2 + rp2}{2}$	$a2 = 198821.001\,km$
Eccentricity	$e2 := \frac{ra2}{a2} - 1$	$e2 = 0.965441$
Orbit Period	$Tp2 := 2 \cdot \pi \cdot \sqrt{\frac{a2^3}{\mu}}$	$Tp2 = 245.077\,hr$
Mean Motion	$n2 := \frac{2 \cdot \pi}{Tp2}$	$n2 = 0.097928569\,\frac{rev}{day}$
Specific orbital energy	$E2 := \frac{-\mu}{2 \cdot a2}$	$E2 = -1.002 \cdot 10^6\,\frac{J}{kg}$
Velocity at Apogee	$va2 := Velo(E2, ra2)$	$va2 = 187.753\,m \cdot s^{-1}$
Velocity at Perigee	$vp2 := Velo(E2, rp2)$	$vp2 = 10677.976\,m \cdot s^{-1}$

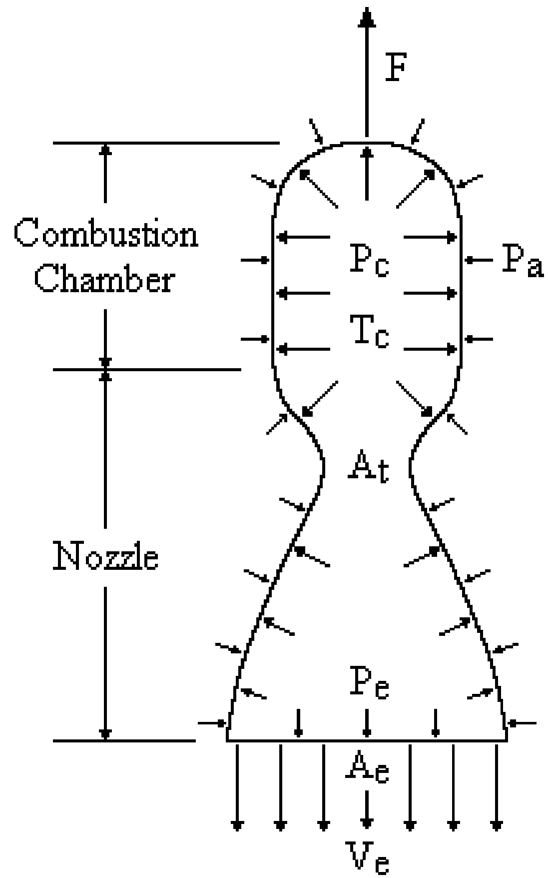
How Does a Rocket Work ???

No, it does not push against the surrounding atmosphere !!!

The Chinese were the first to use solid fuel rockets in the siege of Peiping in 1232.

Newton provided in 1687 the theoretical foundation for understanding how a rocket works.

The bombardment of Copenhagen by the British navy in 1805 was the first large scale military application of rockets in Europe



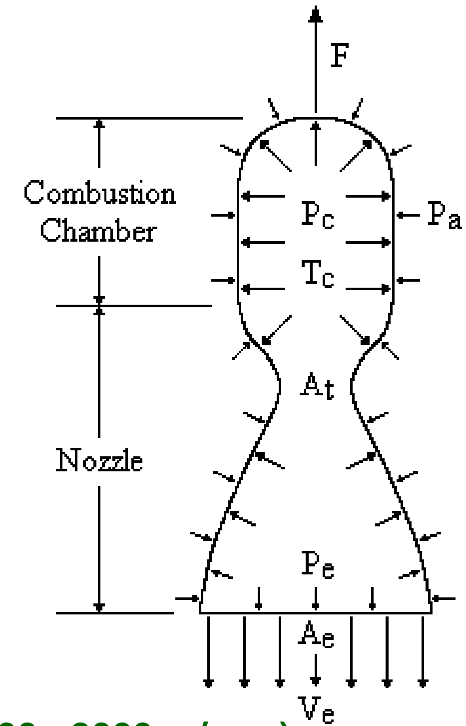
♦ Isaac Newton (1642 - 1727)

- **Philosophiae Naturalis Principia Mathematica 1687**
- 1st Law: The law of Inertia
- 2nd Law: Force = mass x acceleration
- 3rd Law: Action og reaction

Rocket Equation

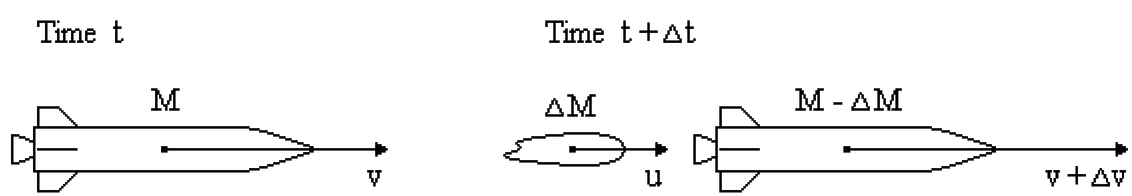
Impulse: $p = m \cdot v$

Conservation of Impulse:
$\Delta M \cdot v_e = M \cdot \Delta v$



$$\textbf{Thrust: } F = q \cdot V_e + (P_e - P_a) \cdot A_e = q \cdot g_0 \cdot I_{sp} + (P_e - P_a) \cdot A_e$$

- g_0: Earth gravitational acceleration
- I_{sp}: Specific Impulse
- A_e: Nozzle exit area
- P_a: Ambient pressure
- P_e: Exit pressure of combustion gases
- V_e: Relative velocity of combustion gases (typically 2500 - 3000 m/sec.)
- q: Expelled mass, kg/sec.



(a) (b)

DSRI

Data for Some Typical Rocket Fuels

Liquid Oxygen / Liquid Hydrogen:	Isp = 455 sec. (Space Shuttle Main Engine)
Liquid Oxygen / Kerosene (RP-1):	Isp = 350 sec.
Solid fuel - Aluminium powder + Ammonium Perchlorate (NH_4ClO_4) + synthetic rubber:	Isp = 280 – 300 sec.
Bi-Propellant Monomethyl Hydrazine (CH_3NHNH_2) + Nitrogen Tetroxide (N_2O_4)	Isp = 300 – 340 sec.
Catalythic Decomposition of Hydrogen Peroxide (H_2O_2)	Isp = 150 sec.
Catalythic Decomposition of Nitrous Oxide (N_2O) (Laughing Gas)	Isp = 150 sec
Cold Gas (N_2, NH_3, He, Freon)	Isp = 50 – 75 sec.

Earth Gravity Acceleration: ge = 9.80665 m/s^2

Ariane 5 – Europe's New Launcher

Ariane 5 – Some Interesting Numbers

Numbers from our Surroundings

- Kitchen tap fully opened:
  ≈0.25 liters/sec.
- Filling of fuel oil tank: ≈2.5 liter/sek.
- Ordinary oil/gas furnace: ≈18 kW
- Mols-line high-speed ferries:
  30000 HK gas turbine (25 MW)
- Avedøre power station: 250 MWe
- Asnæs power station : 610 MWe
  Typical nuclear power station:
  1 GWe, 3 GWt
- Jet engine (Rolls-Royce Trent 800,
  (2 used in Boeing 777):
  Up til 47 t thrust per motor

Ariane 5

- Acceleration:
  At take-off: ≈0.6 g, max. ≈3 g
- Power:
  EPC: ≈4000 MW
  EAP: ≈21000 MW pr stk
  Totalt: ≈46000 MW = 46 GW

EPS – Upper Stage

- 9.7 t fuel total
- Burn time: ≈1100 sec. (≈18 min.)
- Fuel consumption: 8.8 kg/sec.
- Thrust: 2.8 t

EPC – Main Stage

- 27 t liquid hydrogen (-253 °C, 0.071 kg/liter, 390 m^3)
- 130 t liquid oxygen (-183 °C, 1.14 kg/liter, 115 m^3)
- Burn time: ≈590 sek. (9 min. 50 sec.)
- Fuel consumption :
  Liquid hydrogen: 44 kg/s (625 liters/sec.)
  Liquid oxygen: 228 kg/s (200 liters/sec.)
- Power to H_2 turbo pump: 15800 HK (11.9 MW)
- Power to O_2 turbo pump: 5000 HK (3.74 MW)
- Thrust: 115 t (1130 kN)

EAP – Two Solid Fuel Boosters

- 238 t solid fuel per booster
- Burn time: 130 sec. (2 min. 10 sec.)
- Fuel consumption: 1800 kg/sek. per booster
- Thrust: 610 t per booster (6000 kN)

Fuel, Environmental Effects, Ariane 5 and the Ozone Layer

EPS – Upper Stage

- Hypergolic (self igniting) fuel/oxidizer combnination:
  Monomethyl Hydrazine (CH_3NHNH_2), clear liquid, highly toxic
  Nitrogen Tetroxide (N_2O_4), reddish liquid, highly toxic
- Can be ignited and shut down as needed
- Combustion gases: CO_2, H_2O, nitrous compounds
- Environmeltal effects: None in the biosphere, as this stage burns outside the atnosphere.

EPC - Hovedtrin

- Fuel: Liquid Hydrogen, Oxidizer: Liquid Oxygen
- Combustion gases: Water (H_2O)
- Environmeltal effects: None

EAP - 2 stk faststofraketter

- Fuel: Aluminium powder (68 %) + Ammonium Perchlorate (NH_4ClO_4) (18 %) + Hydroxyl-terminated polybutadiene (synthetic rubber) (14 %)
- The components are mixed in a liquid phase, molded into the rocket segments, solidifies to rubber eraser like consistency
- Combustion gases: Aluminium Oxide (Al_2O_3), Hydro-Chloric gas (HCl), CO_2, nitrous compounds etc.
- Environmeltal effects: Hydro-chloric gas is highly toxic, gives rise to acid rain.
- Impact on Ozone layer: When the rocket traverses the stratosphere, HCl “burns” a temporary ozone hole.
- Total ozone-depleting effect of rocket launches is <0.5% all ozone-depleting gases released.
- Why solid rockets when they are not environmentally safe ??? They are simple, cheap and reliable.!!!

Orbit Calculations, Concluded

Calculation of amount of fuel needed to change apogee for Geostationary Transfer Orbit (GTO) to reach the Moon

E: Specific energy
r: Radius
Isp: Specific impulse for the fuel/oxidizer
m: Initial mass of the rocket
$ge = 9.80665\ m/s^2$

Propulsion Calculation

Delta-V to raise apogee by perigee burn	$\Delta v := \left| vp2 - vp1 \right|$	$\Delta v = 673.532\, m \cdot s^{-1}$
Initial spacecraft mass	$m0 := 100 \cdot kg$	

Solid Propellant

Specific impulse of propellant	$Isp4 := 285 \cdot s$	NB: Realistic Isp value
Mass of propellant	$mp4 := PropMass(m0, \Delta v, Isp4)$	
	$mp4 = 21.403\, kg$	
Total engine mass	$me4 := \frac{mp4}{0.88}$	$me4 = 24.322\, kg$

(Based on Thiokol Star 13B, 47 kg total mass)

Formulas

Velocity vs. radius length in orbit

$$Velo(E, r) \equiv \sqrt{2 \cdot \left(E + \frac{\mu}{r} \right)}$$

Propellant mass for given delta-V

$$PropMass(m, \Delta V, Isp) \equiv m \cdot \left(1 - e^{\frac{-\Delta V}{Isp \cdot ge}} \right)$$

Problem

Assume that the Cubesat is in a 600 km circular orbit around the Earth

Calculate the amount of fuel needed to change the orbit to be elliptic with a perigee of 150 km to ensure fast reentry such that the Cubesat does not contribute to space debris.

Catalythic Decomposition of Nitrous Oxide (N_2O) (Laughing Gas)	**Isp = 150 sec**
Earth Gravity Acceleration:	**ge = 9.80665 m/s^2**
Geocentric Gravitational Constant:	**μ =3.986004418·10^{14} m^3/s^2**
Earth Radius at Equator	**R_{ee} = 6378.137 km (WGS-84 ellipsoid)**

Specific Orbital Energy: $$E := \frac{-\mu}{2 \cdot a}$$

Velocity at apogee or perigee: $$V := \sqrt{2 \cdot \left(E + \frac{\mu}{r}\right)}$$

Propellant Mass: $$\mathrm{PropMass}(m, \Delta V, \mathrm{Isp}) \equiv m \cdot \left(1 - e^{\frac{-\Delta V}{\mathrm{Isp} \cdot ge}}\right)$$