DTU Satellite Systems and Design Course

CubeSat Communication

Flemming Hansen
MScEE, PhD
Technology Manager
Danish Small Satellite Programme
Danish Space Research Institute

Phone: 3532 5721
E-mail: fh@dsri.dk

Downloads available from:
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Communication Fundamentals

Imagine a radio transmitter that emits a power P_t equally in al directions (isotropically).

At distance d from the transmitter the transmitted power is distributed equally on the surface of a sphere with radius d and area $4\pi d^2$.

The flux density in W/m^2 of an isotropically radiating antenna at distance d is therefore:

$$S = \frac{P_t}{4 \cdot \pi \cdot d^2}$$

Transmitting Antenna

Any physical antenna will have some directivity, i.e. ability to concentrate the emitted power In a certain direction. Thus, less than the full surface area of the sphere is "illuminated", i.e. less than 4π steradian of solid angle.

The ratio between the full 4π steradian spherical coverage and the actually illuminated solid angle Ω is called the directivity and assumes that power is evenly distributed over Ω and zero outside.

The "antenna gain" G_t is the ratio of flux density in a specific direction at distance d and the flux density from the same transmitter using a hypothetical isotropic antenna.

Mobile phone with low-gain antenna

High-gain parabolic antenna

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Receiving Antenna

The receiving antenna is assumed to have an "effective area" which "collects" the radio waves by intercepting the flux of electromagnetic energy. This means that the receiving antenna collects the total power $P_r = S \cdot A_r$.

A receiving antenna has an antenna gain in the same way as the transmitting antenna. The relationship between the antenna gain and the effective area is given by:

$$A_r = \frac{\lambda^2}{4 \cdot \pi} \cdot G_r$$

$\lambda = c/f$ is the wavelength of the transmitted signal also known as the carrier

c is the velocity of light (and radio waves) in vacuum, $c = 2.99792458 \cdot 10^8$ m/s.

f is the frequency of the carrier.

It can be shown theoretically that the transmitting and receiving antenna gain is the same for the same antenna at the same frequency.

Parabolic Antenna

The gain of a parabolic antenna very often used in ground stations and sometimes also on satellites is given by:

$$G_a := \eta \cdot \left(\frac{\pi \cdot D}{\lambda} \right)^2$$

η (eta) is the socalled aperture efficiency of the antenna,

D is the diameter of the dish

λ (lambda) is the wavelength of the carrier

For good commercial parabolic antennas the efficiency typically falls in the interval:

$$0.6 \leq \lambda \leq 0.7$$

Link Budget - 1

Fundamentals

The link budget is the foundation for designing any radio link, regardless if is terrestrial or in space.

The above considerations are gathered into a single equation that expresses the relationship between the tramsmitter power and the power at the output terminals of the receivning antenna, collecting the power $P_r = S \cdot A_r$

$$P_r = \frac{G_t \cdot P_t}{4 \cdot \pi \cdot d^2} \cdot A_r = \frac{G_t \cdot P_t}{4 \cdot \pi \cdot d^2} \cdot \frac{\lambda^2}{4 \cdot \pi} \cdot G_r$$

$$= \left(\frac{\lambda}{4 \cdot \pi \cdot d} \right)^2 \cdot P_t \cdot G_t \cdot G_r = \left(\frac{c}{4 \cdot \pi \cdot d \cdot f} \right)^2 \cdot P_t \cdot G_t \cdot G_r$$

Link Budget - 2

Path Loss

The quantity $(\lambda/4\pi d)^2$ eller $(c/4\pi df)^2$ is also denoted L_p^{-1} and ic called the path attenuation or path loss), i.e.:

$$L_p = \left(\frac{4 \cdot \pi \cdot d \cdot f}{c} \right)^2$$

Note that L_p is a dimensionless quantity

The relationship between the received and the transmitted powers may now simply be expressed as

$$P_r/P_t = G_t \cdot G_r/L_p$$

Link Budget - 3

dB Calculation

**Telecommunications engineers like calculations in dB (decibal)
Kommmunikationsingeniører kan godt lide at regne i dB eller decibel.**

This is a logarithmic measure expressed fundamentally as the ratio: $10 \cdot \log(P_1/P_2)$

where P_1 and P_2 are powers.

A power ratio of 10 becomes 10 dB, A power ratio of 2 becomes 3 dB (more accurately 3.01 dB, but in everyday jargon 3 dB).

Every time the power ratio is increased by a factor 10, 10 dB is added.

Reducing by a factor 10, subtracts 10 dB

A doubling adds 3 dB, halving subtracts 3 dB etc.

This implies that multiplication transforms into addition and divsion into subtraction, i.e. the link budget calculations are no more complex than checking the bill from the super market !!!

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Link Budget - 4

The Link Budget in dB

Taking the link budget equation and subjecting it to the dB-transformation yields:

$$10\cdot\log(P_r/P_t) = 10\cdot\log(G_t\cdot G_r/L_p)\ \text{[dB]} \qquad \Leftrightarrow$$

$$10\cdot\log(P_r) - 10\cdot\log(P_t) = 10\cdot\log(G_t) + 10\cdot\log(G_r) - 10\cdot\log(L_p)\ \ \text{[dB]}$$

↑↑↑↑↑↑↑↑↑↑↑ **math violation !!!!!**

We cannot just take the log of a dimensioned quantity, here Watts, the P's.

Therefore, we must select a reference power, e.g. 1 Watt and divide the P's by this quantity, so we don't violate the math.

Using this trick and writing the link budget equation again yields:

$$10\cdot\log(P_r/1\ \text{W}) - 10\cdot\log(P_t/1\text{W}) = 10\cdot\log(G_t) + 10\cdot\log(G_r) - 10\cdot\log(L_p)\ \ \text{[dB]}$$

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Link Budget - 5

The Link Budget in dB – Skipping the logs

We want to get rid of the 10·log(*) in all calculations (as far as possible) and switch now entirely to the ”dB-domain” and ”recycle” the letter designations:

$$P_r - P_t = G_t + G_r - L_p \quad [dB] \qquad \Leftrightarrow \qquad P_r = P_t + G_t + G_r - L_p \quad [dB]$$

To interpret power quantities correctly we need a nomanclature telling us the reference value used.

Having used 1 W as a reference above we write ”dBW” instead of just dB, when giving a dB-value for the power

1 W corresponds to 0 dBW, 2 W to 3 dBW, 10 W to 10 dBW etc.

In many cases 1 mW is used as a reference, writing then: “dBm” instead of “dBW”.

Note that dBm og dBW is only about powers. Dimensionless quantities like gain and attenuation just uses dB.

Link Budget - 6

The Link Budget Concluded

The path attenuation may be transformed into the dB-domain as well: $L_p = \left(\frac{4 \cdot \pi \cdot d \cdot f}{c}\right)^2$

and transforms into:

$$L_p = 32.45 + 20 \cdot \log\left(\frac{d}{1 \cdot km}\right) + 20 \cdot \log\left(\frac{f}{1 \cdot MHz}\right)$$

where the 32.45 dB contains the constant 4π/c as well as the powers of 10 coming from using kilometer instead of meter for the distance and Megahertz instead of Hertz for the carrier frequency. We cannot skip the logs entirely:

$$P_r := P_t + G_t + G_r - 32.45 - 20 \cdot \log\left(\frac{d}{1 \cdot km}\right) - 20 \cdot \log\left(\frac{f}{1 \cdot MHz}\right)$$

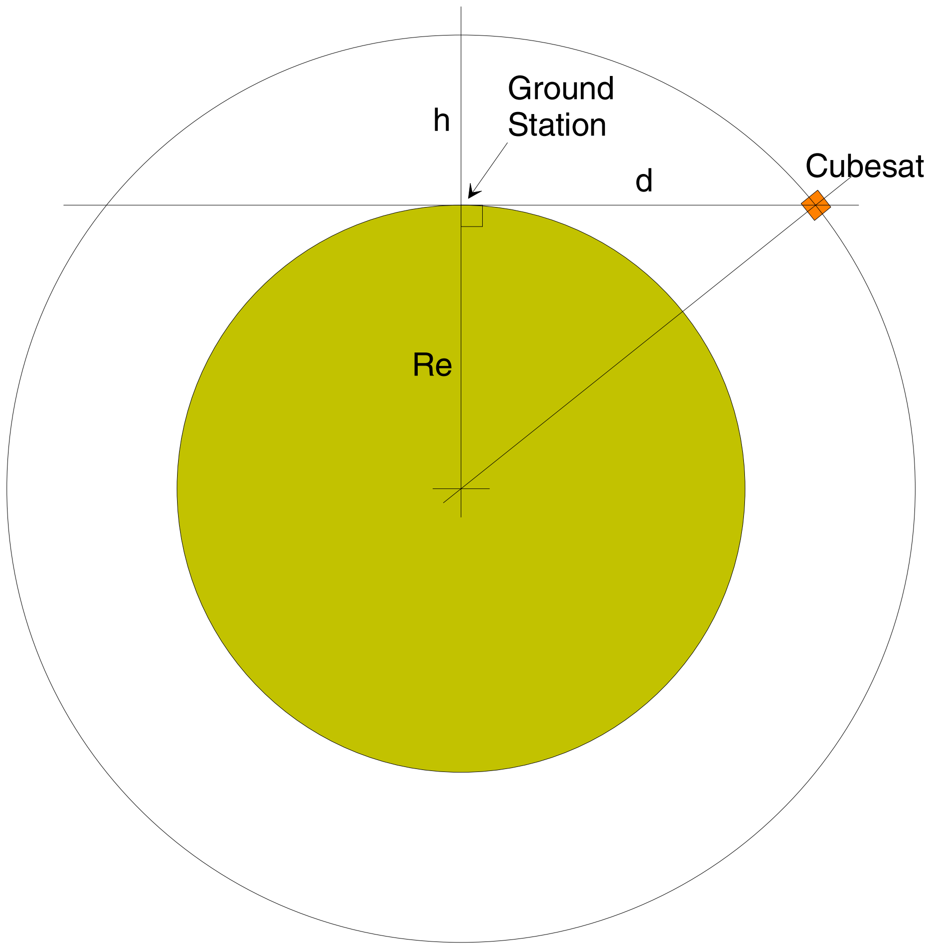
Problem # 1

Distance to Satellite at Horizon

Assume our Cubesat in a h = 600 km perfectly circular orbit

Assume that the Earth is perfecly spherical with a radius Re = 6378 km

Calculate the distance d to the satellite when it is at the geometrical horizon seen from the Ground Station



Solution

Using Pythagoras on the triangle Geocenter – Ground Station - Cubesat yields:

$$d := \sqrt{2 \cdot R_e \cdot h + h^2} = 2830.830 \text{ km}$$

Problem # 2

Path Loss

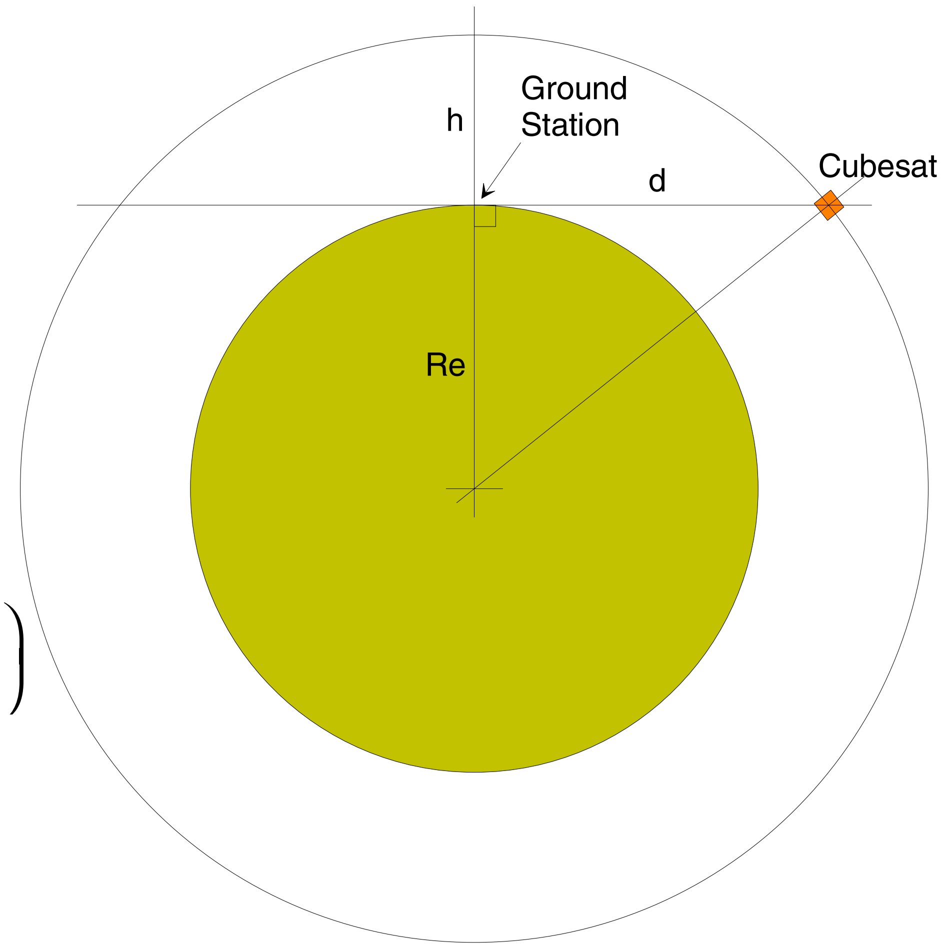
Calculate the path loss for the radio link to the satellite at horizon using the following frequencies:

f_1 = 145.8 MHz (typical amateur sat. uplink)

f_2 = 2215 MHz (Ørsted downlink)

Path loss formula:

$$L_p = 32.45 + 20 \cdot \log\left(\frac{d}{1 \cdot km}\right) + 20 \cdot \log\left(\frac{f}{1 \cdot MHz}\right)$$



Solution

L_{p1} = 144.76 dB

L_{p1} = 168.40 dB

Thermal Noise - 1

Everywhere Noise

Any electronic circuit produces noise, thus limiting hoe small a signal we may amplify and detect. Even the most simple electronic component, a resistor, creates noise.

Additionally, noise will be captured by an antenna looking into space from the cosmic background radiation and from the attenuation of the radio waves passing through the atmosphere caused by water vapour.

Noise calculations again rely on power considerations. Using the theory of thermodynamics it can be shown that an ideal ohmic resistor in thermal equilibrium at absolute temperature T will produce an ”available noise power” P_n given by

$$P_n = kTW = N_0 \cdot W \quad \text{where } N_o = kT$$

k is Boltzmann's constant $k = 1.38062 \cdot 10^{-23}$ J/K

W is the width of the frequency band containing our signal

N_0 is the noise spectral density, has the unit Watt per Hertz of bandwidth and denotes the noise power available in a 1 Hz band. In many cases N_0 is a constant

Thermal Noise - 2

System Noise Temperature

In many cases N_0 is a constant regardless of frequency and the noise thus denoted ”white noise” in analogy to white light that contains an equal amount of light at all wavelengths.

The noise in a radio receiver consists of many different contributions. These are often all referred to an interface at the antenna terminals including the noise from the antenna itself.

All noise contributions are espressed as ”noise tempeartures” and the sum is denoted the ”system noise temperature” T_{sys} or just T for short.

The total noise power at the receiver input is therefore:

$$P_n = P_{n,sys} = kT_{sys} \cdot W = kTW$$

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Thermal Noise - 3

Signal to Noise Ratio

The Signal-to-Noise Ratio – SNR is the ratio between the power of the information carrying signal and the power of the unwanted noise (in the same bandwidth):

$$\Re = P_s/P_n$$

where P_s is the signal power and P_n is the noise power.

In the dB-domain this comes out as:

$$SNR = P_s - P_n$$

Using the link budget equation: $P_r = P_t + G_t + G_r - L_p$ [dB] prevoiusly derived, we get:

$$SNR = P_t + G_t + G_r - L_p - P_n$$

The value of the SNR is the determining factor whether we can extract useful information from the received radio signal or not.

Digital Communication - 1

Modulation

A radio signal, a modulated carrier is a sinusoidal alternating voltage, normally expressed using cosine:

$$S(t) = A \cdot \cos(\omega \cdot t + \varphi) = A \cdot \cos(2 \cdot \pi \cdot f \cdot t + \varphi)$$

where t er time, A signal amplitude, ω (omega) the angular frequency of the carrier $\omega = 2 \cdot \pi \cdot f$, where f is the carrier frequency

φ (phi) is the phase $0° \leq \varphi < 360°$ or $0 \leq \varphi < 2\pi$ radian.

The process of transferring information to a carrier is denoted modulation and involves varying one of the three parameters

A amplitude modulation - AM

ω frequency modulation - FM

φ phase modulation - PM.

The most common format for space communications is phase modulation – PM.

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Digital Communication - 2

Digital Modulation

If we imagine our digital information as a bit stream of speed B bits per second, the duration of each bit is

$$\tau = 1/B \quad [\text{seconds}]$$

Phase Modulation - PM

Let the phase 0° represent binary 0 og phasen 180° represent binary 1.

Hold the phase of the carrier for τ seconds, after which we take the next bit period

This format is denoted “Phase Shift Keying – PSK” or BPSK, where B means Binary

Using simple arguments it is easy to realize that PSK using phases 0° and 180° is identical to amplitude modulation using the amplitdes +1 and –1. Try it yourself:

$$S(t) = A \cdot \cos(\omega \cdot t + \varphi) = A \cdot \cos(2 \cdot \pi \cdot f \cdot t + \varphi)$$

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Digital Communication - 3

The Link Budget and Signal-to-Noise Ratio Revisited

We may now introduce the universal Signal-to-Noise Ratio for digital communication:

$$E_b/N_0 = P_r \cdot \tau/N_0 = P_r/(B \cdot N_0) = P_r/(B \cdot k \cdot T)$$

This is denoted "E-b-over-N-zero" or "Ebno" for short

It expresses the energy per bit divided by the noise spectral density

Examination of the Parameters Used

Energy per bit at the receiver input is the received power P_r times the bit period τ (Watt·seconds = Joule/sec · sec = Joule).

The denominator $B \cdot N_0$ may be interpreted as the noise power in a bandwidth B corresponding to the bit rate in Hertz.

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Digital Communication - 4

To proceed, we rewrite the "Ebno" equation in the dB-domain:

$$10 \cdot \log(E_b/N_0) = 10 \cdot \log(P_r) - 10 \cdot \log(B/1\ \text{Hz}) - 10 \cdot \log(k/1\ \text{J/K}) - 10 \cdot \log(T/1\ \text{K}) \quad [\text{dB}]$$

or

$$E_b/N_0 = P_r - 10 \cdot \log(B/1\ \text{Hz}) - 10 \cdot \log(k/1\ \text{J/K}) - 10 \cdot \log(T/1\ \text{K}) \quad [\text{dB}]$$

where E_b/N_0 is now expressed in dB and P_r in dBW or dBm. As usual proper reference values must be used to make the arguments to the logs dimensionless.

We complete ignore the inconsistency of writing E_b/N_0 in the dB expression. E_b/N_0 is now a symbol, not a calculation.

Combining this with the link budget equation $P_r = P_t + G_t + G_r - L_p$ [dB], we get:

$$E_b/N_0 = (P_t + G_t) + G_r - L_p - 10 \cdot \log(B/1\ \text{Hz}) - 10 \cdot \log(k/1\ \text{J/K}) - 10 \cdot \log(T/1\ \text{K}) \quad [\text{dB}]$$

The quantity $P_t + G_t$ is denoted EIRP (Equivalent Isotropically Radiated Power):

$$E_b/N_0 = \text{EIRP} + G_r - L_p - 10 \cdot \log(B/1\ \text{Hz}) - 10 \cdot \log(k/1\ \text{J/K}) - 10 \cdot \log(T/1\ \text{K}) \quad [\text{dB}]$$

Digital Communication - 5

We now introduce the path loss equation:

$$L_p = 32.45 + 20 \cdot \log\left(\frac{d}{1 \cdot km}\right) + 20 \cdot \log\left(\frac{f}{1 \cdot MHz}\right)$$

into the Ebno equation and get:

$$E_b/N_0 = EIRP + G_r - 32.45 - 20 \cdot \log(d/1\ km) - 20 \cdot \log(f/1\ MHz) - 10 \cdot \log(B/1\ Hz) - 10 \cdot \log(k/1\ J/K) - 10 \cdot \log(T/1\ K) \quad [dB]$$

The dB-value of Boltzmann's constant is calculated to 10·log(k/1 J/K) = -228.60 dB and introduced into the equation. In addition the equation is restructured a little:

$$E_b/N_0 = EIRP + (G_r - 10 \cdot \log(T/1\ K)) - 32.45 + 228.60 - 20 \cdot \log(d/1\ km) - 20 \cdot \log(f/1\ MHz) - 10 \cdot \log(B/1\ Hz) \quad [dB]$$

/continued...

Digital Communication - 6

Further restructuring of the equation yields:

$$E_b/N_0 = EIRP + G/T - 32.45 + 228.60 - 20 \cdot \log(d/1\ km) - 20 \cdot \log(f/1\ MHz) - 10 \cdot \log(B/1\ Hz)$$

or

$$E_b/N_0 = EIRP + G/T + 196.15 - 20 \cdot \log(d/1\ km) - 20 \cdot \log(f/1\ MHz) - 10 \cdot \log(B/1\ Hz)\ \ [dB]$$

This is the magic link budget equation

EIRP = $P_t + G_t$ is the "Equivalent Isotropically Radiated Power" or the power required by the transmitter output stage if the antenna radiated equally in all directions (isotropically). The advantage of using EIRP is that you may trade antenna gain for transmitter output power for a given EIRP requirement.

G/T = $G_r - 10 \cdot \log(T/1\ K)$ [dB/K], pronounced G-over-T, is a measure of the quality factor or performance of the receiver. G/T allows the link designer to trade receiving antenna gain for system noise temperature with a given G/T requirement. Again there is an inconsistency of nomenclature. G/T is a symbol, not a calculation.

Digital Communication - 7

Bit Error Probability

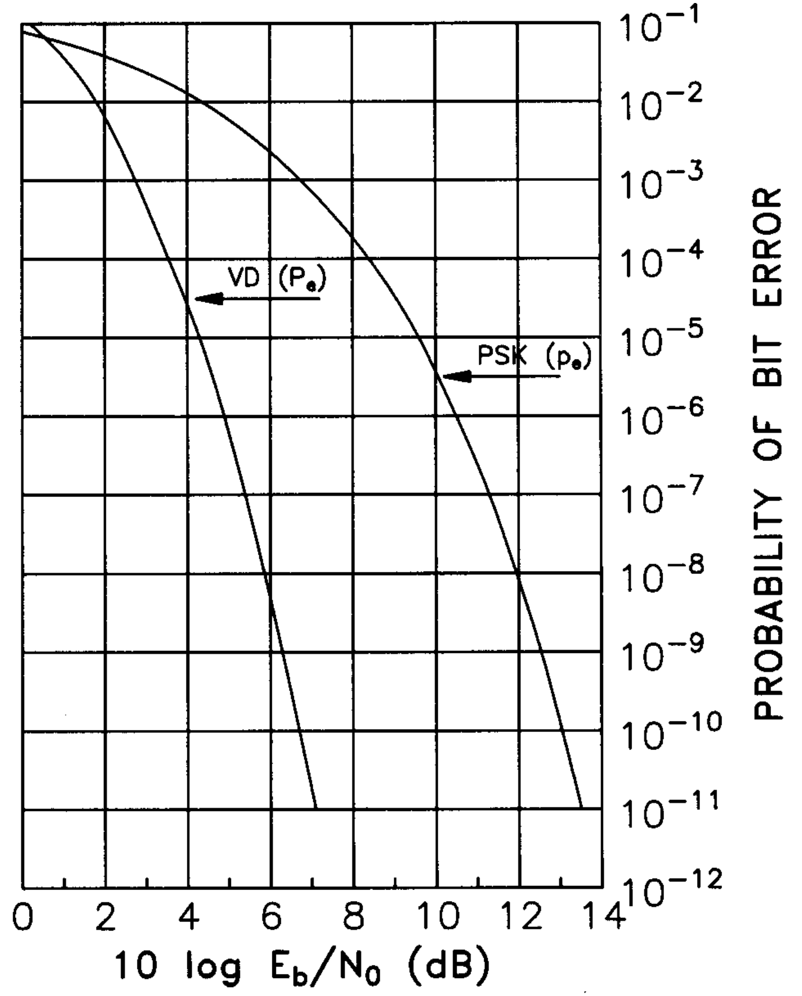
To be able to assess the quality of our radio link a relation between E_b/N_0 and the error rate of the received bits must be established. The bit error rate must be low, but at a reasonable cost only.

The graph marked PSK is a plot of the function

$$P_e := \frac{1}{2} \cdot \mathrm{erfc}\left(\sqrt{\frac{E_b}{N_o}}\right)$$

where Pe is the bit error probability and erfc() is the error function complement, a standard function in probability theory.

The curve marked VD is the bit error probability using forward error correction (FEC) encoding.



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Problem # 3

Cubesat Downlink

Assume that our Cubesat in a 600 km circular orbit has a 1 W (= 0 dBW) transmitter and radiates this power towards the ground station using an antenna with 0 dB gain in the direction of the ground station and the satellite at the horizon.

Carrier frequency: f = 2215 MHz (Ørsted frequency)

Distance to satellite: 2830.830 km at horizon

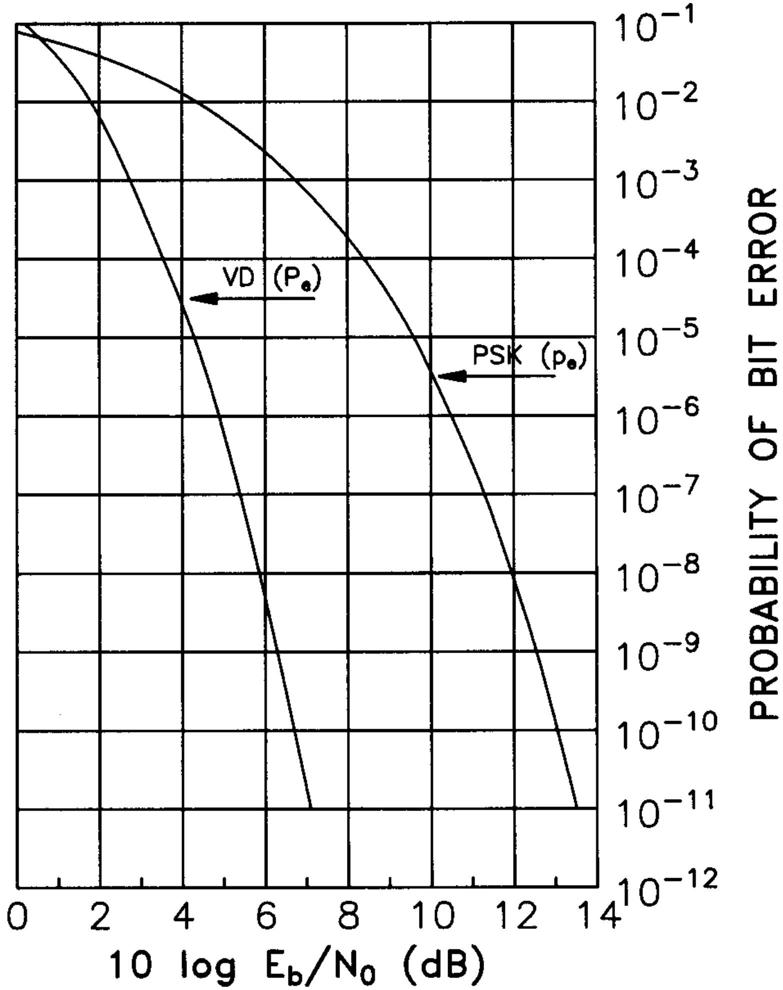
Bit rate: 256 kbit/s

G/T: 5 dB/K (Ørsted ground station at DMI)

Magic link budget equation:

$$E_b/N_0 = EIRP + G/T + 196.15 - 20 \cdot \log(d/1\text{ km}) - 20 \cdot \log(f/1\text{ MHz}) - 10 \cdot \log(B/1\text{ Hz}) \quad [\text{dB}]$$

Calculate E_b/N_0 and read P_e off the graph



Problem # 3 continued

Solution

EIRP = 0 + 0 = 0 dBW

E_b/N_0 = EIRP + G/T + 196.15 - 20·log(d/1 km) - 20·log(f/1 MHz) - 10·log(B/1 Hz) [dB]

E_b/N_0 = 0 + 5 + 196.15 – 20·log(2830.830) - 20·log(2215) - 10·log(256000) [dB]

= 0 + 5 + 196.15 – 69.04 – 66.91 – 54.08 [dB]

= 11.12 dB

$P_e = 1.813 \cdot 10^{-7}$ without forward error correction encoding

Ørsted actually does a lot better as a very efficient FEC encoding is used.